PART 2

Important Financial Concepts

CHAPTERS IN THIS PART

CHAPTER 4

Time Value of Money

INSTRUCTOR'S RESOURCES

Overview

This chapter introduces an important financial concept: the time value of money. The present value and future value of a sum, as well as the present and future values of an annuity, are explained. Special applications of the concepts include intra-year compounding, mixed cash flow streams, mixed cash flows with an embedded annuity, perpetuities, deposits to accumulate a future sum, and loan amortization. Numerous business and personal financial applications are used as examples.

PMF DISK

***PMF Tutor*: Time Value of Money**

Time value of money problems included in the *PMF Tutor* are future value (single amount), present value (single amount and mixed stream), present and future value annuities, loan amortization, and deposits to accumulate a sum.

***PMF Problem-Solver*: Time Value of Money**

This module will allow the student to compute the worth of money under three scenarios: 1) single payment, 2) annuities, 3) mixed stream. These routines may also be used to amortize a loan or estimate growth rates.

PMF Templates

Spreadsheet templates are provided for the following problems:

Problem	Topic
Self-Test 1	Future values for various compounding frequencies
Self-Test 2	Future value of annuities
Self-Test 3	Present value of lump sums and streams
Self-Test 4	Deposits needed to accumulate a future sum

Study Guide

The following *Study Guide* examples are suggested for classroom presentation:

Example	Topic
5	More on annuities
6	Loan amortization
10	Effective rate

ANSWERS TO REVIEW QUESTIONS

4-1 *Future value (FV)*, the value of a present amount at a future date, is calculated by applying compound interest over a specific time period. *Present value (PV)*, represents the dollar value today of a future amount, or the amount you would invest today at a given interest rate for a specified time period to equal the future amount. Financial managers prefer present value to future value because they typically make decisions at time zero, before the start of a project.

4-2 A *single amount* cash flow refers to an individual, stand alone, value occurring at one point in time. An *annuity* consists of an unbroken series of cash flows of equal dollar amount occurring over more than one period. A *mixed stream* is a pattern of cash flows over more than one time period and the amount of cash associated with each period will vary.

4-3 *Compounding* of interest occurs when an amount is deposited into a savings account and the interest paid after the specified time period remains in the account, thereby becoming part of the principal for the following period. The general equation for future value in year n (FV_n) can be expressed using the specified notation as follows:

$$FV_n = PV \times (1+i)^n$$

4-4 A decrease in the interest rate lowers the future amount of a deposit for a given holding period, since the deposit earns less at the lower rate. An increase in the holding period for a given interest rate would increase the future value. The increased holding period increases the future value since the deposit earns interest over a longer period of time.

4-5 The present value, PV, of a future amount indicates how much money today would be equivalent to the future amount if one could invest that amount at a specified rate of interest. Using the given notation, the present value (PV) of a future amount (FV_n) can be defined as follows:

$$PV = FV\left(\frac{1}{(1+i)^n}\right)$$

4-6 An increasing required rate of return would reduce the present value of a future amount, since future dollars would be worth less today. Looking at the formula for present value in question 5, it should be clear that by increasing the i value, which is the required return, the present value interest factor would decrease, thereby reducing the present value of the future sum.

4-7 Present value calculations are the exact inverse of compound interest calculations. Using compound interest, one attempts to find the future value of a present amount; using present value, one attempts to find the present value of an amount to be received in the future.

4-8 An *ordinary annuity* is one for which payments occur at the end of each period. An *annuity due* is one for which payments occur at the beginning of each period.

The ordinary annuity is the more common. For otherwise identical annuities and interest rates, the annuity due results in a higher future value because cash flows occur earlier and have more time to compound.

4-9 The present value of an ordinary annuity, PVA_n, can be determined using the formula:

$$PVA_n = PMT \times (PVIFA_{i\%,n})$$

Where:

PMT	=	the end of period cash inflows
$PVIFA_{i\%,n}$	=	the present value interest factor of an annuity for interest rate i and n periods.

The PVIFA is related to the PVIF in that the annuity factor is the sum of the PVIFs over the number of periods for the annuity. For example, the PVIFA for 5% and 3 periods is 2.723, and the sum of the 5% PVIF for periods one through three is 2.723 (.952 + .907 + .864).

4-10 The FVIFA factors for an ordinary annuity can be converted for use in calculating an annuity due by multiplying the $FVIFA_{i\%,n}$ by 1 + i.

4-11 The PVIFA factors for an ordinary annuity can be converted for use in calculating an annuity due by multiplying the $PVIFA_{i\%,n}$ by 1 + i.

4-12 A *perpetuity* is an infinite-lived annuity. The factor for finding the present value of a perpetuity can be found by dividing the discount rate into 1.0. The resulting quotient represents the factor for finding the present value of an infinite-lived stream of equal annual cash flows.

4-13 The future value of a mixed stream of cash flows is calculated by multiplying each year's cash flow by the appropriate future value interest factor. To find the present value of a mixed stream of cash flows multiply each year's cash flow by the appropriate present value interest factor. There will be at least as many calculations as the number of cash flow.

4-14 As interest is compounded more frequently than once a year, both **(a)** the future value for a given holding period and **(b)** the *effective annual rate* of interest will increase. This is due to the fact that the more frequently interest is compounded, the greater the future value. In situations of intra-year compounding, the actual rate of interest is greater than the stated rate of interest.

4-15 *Continuous compounding* assumes interest will be compounded an infinite number of times per year, at intervals of microseconds. Continuous compounding of a given deposit at a given rate of interest results in the largest value when compared to any other compounding period.

4-16 The *nominal annual rate* is the contractual rate that is quoted to the borrower by the lender. The *effective annual rate*, sometimes called the *true rate*, is the actual rate that is paid by the borrower to the lender. The difference between the two rates is due to the compounding of interest at a frequency greater than once per year.

APR is the *Annual Percentage Rate* and is required by “truth in lending laws” to be disclosed to consumers. This rate is calculated by multiplying the periodic rate by the number of periods in one year. The periodic rate is the nominal rate over the shortest time period in which interest is compounded. The APY, or *Annual Percentage Yield*, is the effective rate of interest that must be disclosed to consumers by banks on their savings products as a result of the “truth in savings laws.” These laws result in both favorable and unfavorable information to consumers. The good news is that rate quotes on both loans and savings are standardized among financial institutions. The negative is that the APR, or lending rate, is a nominal rate, while the APY, or saving rate, is an effective rate. These rates are the same when compounding occurs only once per year.

4-17 The size of the equal annual end-of-year deposits needed to accumulate a given amount over a certain time period at a specified rate can be found by dividing the interest factor for the future value of an annuity for the given interest rate and the number of years ($FVIFA_{i\%,n}$) into the desired future amount. The resulting quotient would be the amount of the equal annual end-of-year deposits required. The future value interest factor for an annuity is used in this calculation:

$$PMT = \frac{FV_n}{FVIFA_{i\%,\,n}}$$

4-18 Amortizing a loan into equal annual payments involves finding the future payments whose present value at the loan interest rate just equals the amount of the initial principal borrowed. The formula is:

$$PMT = \frac{PV_n}{PVIFA_{i\%,\,n}}$$

4-19 **a.** Either the present value interest factor or the future value interest factor can be used to find the growth rate associated with a stream of cash flows.

The growth rate associated with a stream of cash flows may be found by using the following equation, where the growth rate, g, is substituted for k.

$$PV = \frac{FV_n}{(1+g)}$$

To find the rate at which growth has occurred, the amount received in the earliest year is divided by the amount received in the latest year. This quotient is the $PVIF_{i\%;n}$. The growth rate associated with this factor may be found in the PVIF table.

b. To find the interest rate associated with an equal payment loan, the Present Value Interest Factors for a One-Dollar Annuity Table would be used.

To determine the interest rate associated with an equal payment loan, the following equation may be used:

$$PV_n = PMT \times (PVIFA_{i\%,n})$$

Solving the equation for $PVIFA_{i\%,n}$ we get:

$$PVIFA_{i\%,\,n} = \frac{PV_n}{PMT}$$

Then substitute the values for PV_n and PMT into the formula, using the PVIFA Table to find the interest rate most closely associated with the resulting PVIFA, which is the interest rate on the loan.

4-20 To find the number of periods it would take to compound a known present amount into a known future amount you can solve either the present value or future value equation for the interest factor as shown below using the present value:

$$PV = FV \times (PVIF_{i\%,n})$$

Solving the equation for $PVIF_{i\%,n}$ we get:

$$PVIF_{i\%,\,n} = \frac{PV}{FV}$$

Then substitute the values for PV and FV into the formula, using the PVIF Table for the known interest rate find the number of periods most closely associated with the resulting PVIF.

The same approach would be used for finding the number of periods for an annuity except that the annuity factor and the PVIFA (or FVIFA) table would be used. This process is shown below.

$$PV_n = PMT \text{ x } (PVIFA_{i\%,n})$$

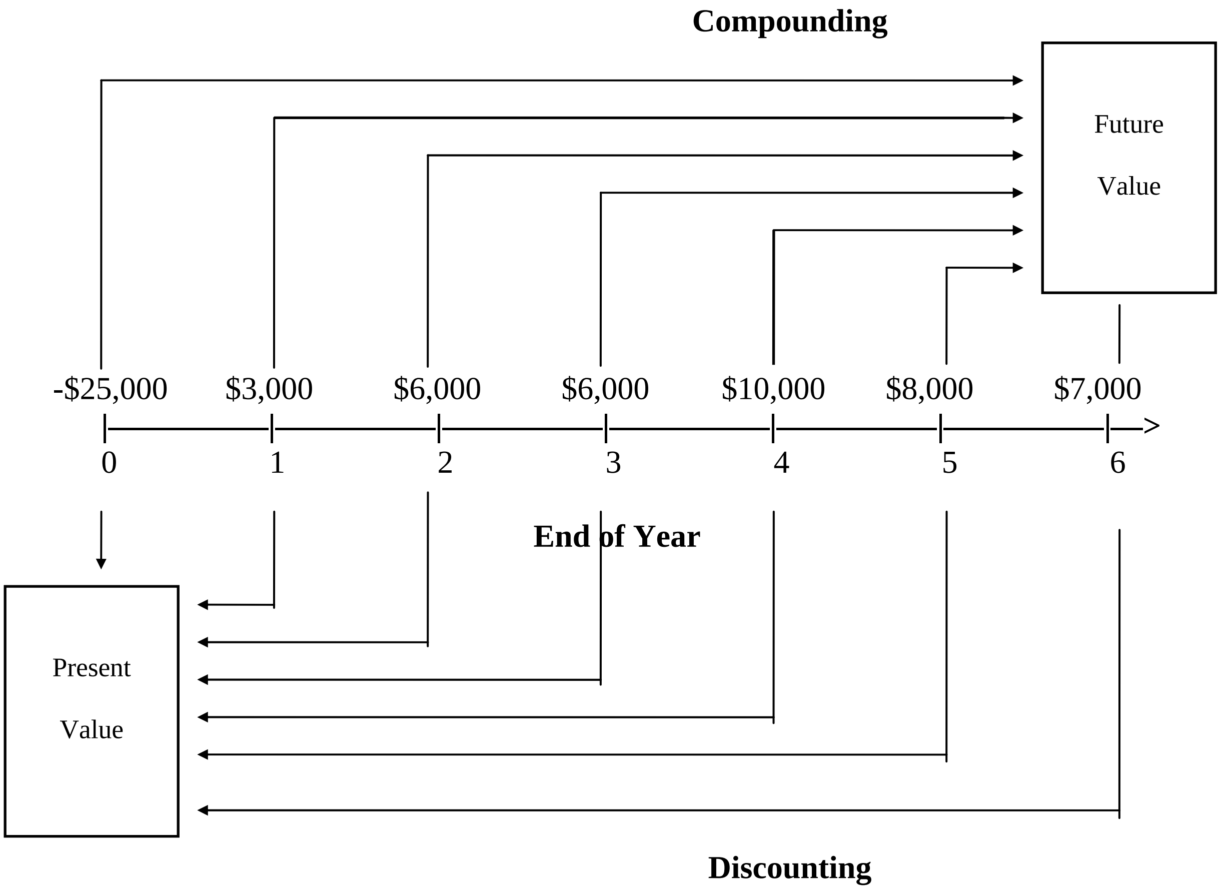
Solving the equation for $PVIFA_{i\%,n}$ we get:

$$PVIFA_{i\%,n} = \frac{PV_n}{PMT}$$

SOLUTIONS TO PROBLEMS

4-1 LG 1: Using a Time Line

a., b., c.



d. Financial managers rely more on present than future value because they typically make decisions before the start of a project, at time zero, as does the present value calculation.

4-2 LG 2: Future Value Calculation: $FV_n = PV \times (1+i)^n$

Case

A $FVIF_{12\%,2\ periods} = (1+.12)^2 = 1.254$

B $FVIF_{6\%,3\ periods} = (1+.06)^3 = 1.191$

C $FVIF_{9\%,2\ periods} = (1+.09)^2 = 1.188$

D $FVIF_{3\%,4\ periods} = (1+.03)^4 = 1.126$

4-3 LG 2: Future Value Tables: $FV_n = PV \times (1+i)^n$

Case A

a.
$2 = 1 \times (1+.07)^n$
$2/1 = (1.07)^n$
$2 = FVIF_{7\%,n}$
10 years< n < 11 years
Nearest to 10 years

b.
$4 = 1 \times (1+.07)^n$
$4/1 = (1.07)^n$
$4 = FVIF_{7\%,n}$
20 years < n < 21 years
Nearest to 20 years

Case B

a. $2 = 1 \times (1 + .40)^n$
$2 = FVIF_{40\%,n}$
2 years < n < 3 years
Nearest to 2 years

b. $4 = (1 + .40)^n$
$4 = FVIF_{40\%,n}$
4 years < n < 5 years
Nearest to 4 years

Case C

a. $2 = 1 \times (1 + .20)^n$
$2 = FVIF_{20\%,n}$
3 years < n < 4 years
Nearest to 4 years

b. $4 = (1 + .20)^n$
$4 = FVIF_{20\%,n}$
7 years < n < 8 years
Nearest to 8 years

Case D

a. $2 = 1 \times (1 +.10)^n$
$2 = FVIF_{10\%,n}$
7 years < n < 8 years
Nearest to 7 years

b. $4 = (1 +.10)^n$
$4 = FVIF_{40\%,n}$
14 years < n <15 years
Nearest to 15 years

4-4 LG 2: Future Values: $FV_n = PV \times (1 + i)^n$ or $FV_n = PV \times (FVIF_{i\%,n})$

Case

A $FV_{20} = PV \times FVIF_{5\%,20\text{ yrs.}}$
$FV_{20} = \$200 \times (2.653)$
$FV_{20} = \$530.60$
Calculator solution: $530.66

B $FV_7 = PV \times FVIF_{8\%,7\text{ yrs.}}$
$FV_7 = \$4{,}500 \times (1.714)$
$FV_7 = \$7{,}713$
Calculator solution; $7,712.21

C $FV_{10} = PV \times FVIF_{9\%,10\text{ yrs.}}$
$FV_{10} = \$\ 10{,}000 \times (2.367)$
$FV_{10} = \$23{,}670$
Calculator solution: $23,673.64

D $FV_{12} = PV \times FVIF_{10\%,12\text{ yrs.}}$
$FV_{12} = \$25{,}000 \times (3.138)$
$FV_{12} = \$78{,}450$
Calculator solution: $78,460.71

E $FV_5 = PV \times FVIF_{11\%,5\text{ yrs.}}$
$FV_5 = \$37{,}000 \times (1.685)$
$FV_5 = \$62{,}345$
Calculator solution: $62,347.15

F $FV_9 = PV \times FVIF_{12\%,9\text{ yrs.}}$
$FV_9 = \$40{,}000 \times (2.773)$
$FV_9 = \$110{,}920$
Calculator solution: $110,923.15

4-5 LG 2: Future Value: $FV_n = PV \times (1 + i)^n$ or $FV^n = PV \times (FVIF_{i\%,n})$

a

1. $FV_3 = PV \times (FVIF_{7\%,3})$
$FV_3 = \$1{,}500 \times (1.225)$
$FV_3 = \$1{,}837.50$
Calculator solution: $1,837.56

2. $FV_6 = PV \times (FVIF_{7\%,6})$
$FV_6 = \$1{,}500 \times (1.501)$
$FV_6 = \$2{,}251.50$
Calculator solution: $2,251.10

3. $FV_9 = PV \times (FVIF_{7\%,9})$
$FV_9 = \$1{,}500 \times (1.838)$
$FV_9 = \$2{,}757.00$

b.

1. Interest earned = FV_3 - PV
Interest earned = $1,837.50
-$1,500.00
$ 337.50

2. Interest earned = $FV_6 – FV_3$
Interest earned = $2,251.50
-$1,837.50
$ 414.00

3. Interest earned = $FV_9 – FV_6$
Interest earned = $2,757.00
-$2,251.50

Calculator solution: \$2,757.69 $\underline{\underline{\$\quad 505.50}}$

c. The fact that the longer the investment period is, the larger the total amount of interest collected will be, is not unexpected and is due to the greater length of time that the principal sum of \$1,500 is invested. The most significant point is that the incremental interest earned per 3-year period increases with each subsequent 3 year period. The total interest for the first 3 years is \$337.50; however, for the second 3 years (from year 3 to 6) the additional interest earned is \$414.00. For the third 3-year period, the incremental interest is \$505.50. This increasing change in interest earned is due to compounding, the earning of interest on previous interest earned. The greater the previous interest earned, the greater the impact of compounding.

4-6 LG 2: Inflation and Future Value

a.

1. $FV_5 = PV \times (FVIF_{2\%,5})$
$FV_5 = \$14,000 \times (1.104)$
$FV_5 = \$15,456.00$
Calculator solution: \$15,457.13

2. $FV_5 = PV \times (FVIF_{4\%,5})$
$FV_5 = \$14,000 \times (1.217)$
$FV_5 = \$17,038.00$
Calculator solution: \$17,033.14

b. The car will cost \$1,582 more with a 4% inflation rate than an inflation rate of 2%. This increase is 10.2% more (\$1,582 ÷ \$15,456) than would be paid with only a 2% rate of inflation.

4-7 LG 2: Future Value and Time

Deposit now:
$FV_{40} = PV \times FVIF_{9\%,40}$
$FV_{40} = \$10,000 \times (1.09)^{40}$
$FV_{40} = \$10,000 \times (31.409)$
$FV_{40} = \$314,090.00$
Calculator solution: $314,094.20

Deposit in 10 years:
$FV_{30} = PV_{10} \times (FVIF_{9\%,30})$
$FV_{30} = PV_{10} \times (1.09)^{30}$
$FV_{30} = \$10,000 \times (13.268)$
$FV_{30} = \$132,680.00$
Calculator solution: $132,676.79

You would be better off by $181,410 ($314,090 - $132,680) by investing the $10,000 now instead of waiting for 10 years to make the investment.

4-8 LG 2: Future Value Calculation: $FV_n = PV \times FVIF_{i\%,n}$

a. $\$15,000 = \$10,200 \times FVIF_{i\%,5}$
$FVIF_{i\%,5} = \$15,000 \div \$10,200 = 1.471$
$8\% < i < 9\%$
Calculator Solution: 8.02%

b. $\$15,000 = \$8,150 \times FVIF_{i\%,5}$
$FVIF_{i\%,5} = \$15,000 \div \$8,150 = 1.840$
$12\% < i < 13\%$
Calculator Solution: 12.98%

c. $\$15,000 = \$7,150 \times FVIF_{i\%,5}$
$FVIF_{i\%,5} = \$15,000 \div \$7,150 = 2.098$
$15\% < i < 16\%$
Calculator Solution: 15.97%

4-9 LG 2: Single-payment Loan Repayment: $FV_n = PV \times FVIF_{i\%,n}$

a. $FV_1 = PV \times (FVIF_{14\%,1})$
$FV_1 = \$200 \times (1.14)$
$FV_1 = \$228$
Calculator Solution: $228

b. $FV_4 = PV \times (FVIF_{14\%,4})$
$FV_4 = \$200 \times (1.689)$
$FV_4 = \$337.80$
Calculator solution: $337.79

c. $FV_8 = PV \times (FVIF_{14\%,8})$
$FV_8 = \$200 \times (2.853)$
$FV_8 = \$570.60$
Calculator Solution: $570.52

4-10 LG 2: Present Value Calculation: $PVIF = \dfrac{1}{(1+i)^n}$

Case					
A	PVIF	=	$1 \div (1 + .02)^4$	=	.9238
B	PVIF	=	$1 \div (1 + .10)^2$	=	.8264
C	PVIF	=	$1 \div (1 + .05)^3$	=	.8638
D	PVIF	=	$1 \div (1 + .13)^2$	=	.7831

4-11 LG 2: Present Values: $PV = FV_n \times (PVIF_{i\%,n})$

Case			Calculator Solution
A	$PV_{12\%,4yrs}$	= \$ 7,000 x .636 = \$ 4,452	\$ 4,448.63
B	$PV_{8\%,\ 20yrs}$	= \$ 28,000 x .215= \$ 6,020	\$ 6,007.35
C	$PV_{14\%,12yrs}$	= \$ 10,000 x .208= \$ 2,080	\$ 2,075.59
D	$PV_{11\%,6yrs}$	= \$150,000 x .535= \$80,250	\$80,196.13
E	$PV_{20\%,8yrs}$	= \$ 45,000 x .233= \$10,485	\$10,465.56

4-12 LG 2: Present Value Concept: $PV_n = FV_n \times (PVIF_{i\%,n})$

a. PV = $FV_6 \times (PVIF_{12\%,6})$
PV = \$6,000 x (.507)
PV = \$3,042.00
Calculator solution: \$3,039.79

b. PV = $FV_6 \times (PVIF_{12\%,6})$
PV = \$6,000 x (.507)
PV = \$3,042.00
Calculator solution: \$3,039.79

c. PV = $FV_6 \times (PVIF_{12\%,6})$
PV = \$6,000 x (.507)
PV = \$3,042.00
Calculator solution: \$3,039.79

d. The answer to all three parts are the same. In each case the same questions is being asked but in a different way.

4-13 LG 2: Present Value: $PV = FV_n \times (PVIF_{i\%,n})$

Jim should be willing to pay no more than \$408.00 for this future sum given that his opportunity cost is 7%.

PV = \$500 x $(PVIF_{7\%,3})$
PV = \$500 x (.816)
PV = \$408.00
Calculator solution: \$408.15

4-14 LG 2: Present Value: $PV = FV_n \times (PVIF_{i\%,n})$

PV = \$100 x $(PVIF_{8\%,6})$
PV = \$100 x (.630)
PV = \$63.00
Calculator solution: \$63.02

4-15 LG 2: Present Value and Discount Rates: $PV = FV_n \times (PVIF_{i\%,n})$

a. **(1)** PV = \$1,000,000 x $(PVIF_{6\%,10})$
PV = \$1,000,000 x (.558)
PV = \$558,000.00
Calculator solution: \$558,394.78

(2) PV = \$1,000,000 x $(PVIF_{9\%,10})$
PV = \$1,000,000 x (.422)
PV = \$422,000.00
Calculator solution: \$422,410.81

(3) PV = $1,000,000 x (PVIF$_{12\%,10}$)
PV = $1,000,000 x (.322)
PV = $322,000.00
Calculator solution: $321,973.24

b. **(1)** PV = $1,000,000 x (PVIF$_{6\%,15}$)
PV = $1,000,000 x (.417)
PV = $417,000.00
Calculator solution: $417,265.06

(2) PV = $1,000,000 x (PVIF$_{9\%,15}$)
PV = $1,000,000 x (.275)
PV = $275,000.00
Calculator solution: $274,538.04

(3) PV = $1,000,000 x (PVIF$_{12\%,15}$)
PV = $1,000,000 x (.183)
PV = $183,000.00
Calculator solution: $182,696.26

c. As the discount rate increases, the present value becomes smaller. This decrease is due to the higher opportunity cost associated with the higher rate. Also, the longer the time until the lottery payment is collected, the less the present value due to the greater time over which the opportunity cost applies. In other words, the larger the discount rate and the longer the time until the money is received, the smaller will be the present value of a future payment.

4-16 LG 2: Present Value Comparisons of Lump Sums: PV = FV$_n$ x (PVIF$_{i\%,n}$)

a. **A.** PV = $28,500 x (PVIF$_{11\%,3}$)
PV = $28,500 x (.731)
PV = $20,833.50
Calculator solution: $20,838.95

B. PV = $54,000 x (PVIF$_{11\%,9}$)
PV = $54,000 x (.391)
PV = $21,114.00
Calculator solution: $21,109.94

C. PV = $160,000 x (PVIF$_{11\%,20}$)
PV = $160,000 x (.124)
PV = $19,840.00
Calculator solution: $19,845.43

b. Alternatives A and B are both worth greater than $20,000 in term of the present value.

c. The best alternative is B because the present value of B is larger than either A or C and is also greater than the $20,000 offer.

4-17 LG 2: Cash Flow Investment Decision: PV = FV$_n$ x (PVIF$_{i\%,n)}$

A. PV = $30,000 x (PVIF$_{10\%,5}$)
PV = $30,000 x (.621)
PV = $18,630.00
Calculator solution: $18,627.64

B. PV = $3,000 x (PVIF$_{10\%,20}$)
PV = $3,000 x (.149)
PV = $447.00
Calculator solution: $445.93

C. PV = $10,000 x (PVIF$_{10\%,10}$)
PV = $10,000 x (.386)
PV = $3,860.00
Calculator solution: $3,855.43

D. PV = $15,000 x (PVIF$_{10\%,40}$)
PV = $15,000 x (.022)
PV = $330.00
Calculator solution: $331.42

Purchase	Do Not Purchase
A	B
C	D

4-18 LG 3: Future Value of an Annuity

a. Future Value of an Ordinary Annuity vs. Annuity Due

	(1) Ordinary Annuity	**(2) Annuity Due**
	$FVA_{k\%,n}$ = PMT x ($FVIFA_{k\%,n}$)	FVA_{due} = PMT x [($FVIFA_{k\%,n}$ x (1 + k)]
A	$FVA_{8\%,10}$ = \$2,500 x 14.487	FVA_{due} = \$2,500 x (14.487 x 1.08)
	$FVA_{8\%,10}$ = \$36,217.50	FVA_{due} = \$39,114.90
	Calculator solution: \$36,216.41	Calculator solution: \$39,113.72
B	$FVA_{12\%,6}$ = \$500 x 8.115	FVA_{due} = \$500 x(8.115 x 1.12)
	$FVA_{12\%,6}$ = \$4,057.50	FVA_{due} = \$4,544.40
	Calculator solution: \$4,057.59	Calculator solution: \$4,544.51
C	$FVA_{20\%,5}$ = \$30,000 x 7.442	FVA_{due} = \$30,000 x (7.442 x 1.20)
	$FVA_{20\%,5}$ = \$223,260	FVA_{due} = \$267,912
	Calculator solution: \$223,248	Calculator solution: \$267,897.60
	(1) Ordinary Annuity	**(2) Annuity Due**
D	$FVA_{9\%,8}$ = \$11,500 x 11.028	FVA_{due} = \$11,500 x (11.028 x 1.09)
	$FVA_{9\%,8}$ = \$126,822	FVA_{due} = \$138,235.98
	Calculator solution: \$126,827.45	Calculator solution: \$138,241.92
E	$FVA_{14\%,30}$ = \$6,000 x 356.787	FVA_{due} = \$6,000 x (356.787 x 1.14)
	$FVA_{14\%,30}$ = \$2,140,722	FVA_{due} = \$2,440,422.00
	Calculator solution: \$2,140,721.10	Calculator solution: \$2,440,422.03

b. The annuity due results in a greater future value in each case. By depositing the payment at the beginning rather than at the end of the year, it has one additional year of compounding.

4-19 LG 3: Present Value of an Annuity: $PV_{n\,=}$ PMT x ($PVIFA_{i\%,n}$)

a. Present Value of an Ordinary Annuity vs. Annuity Due

	(1) Ordinary Annuity	**(2) Annuity Due**
	$PVA_{k\%,n}$ = PMT x ($PVIFA_{i\%,n}$)	PVA_{due} = PMT x [($PVIFA_{i\%,n}$ x (1 + k)]
A	$PVA_{7\%,3}$ = \$12,000 x 2.624	PVA_{due} = \$12,000 x (2.624 x 1.07)
	$PVA_{7\%,3}$ = \$31,488	PVA_{due} = \$33,692
	Calculator solution: \$31,491.79	Calculator solution: \$33,696.22
B	$PVA_{12\%15}$ = \$55,000 x 6.811	PVA_{due} = \$55,000 x (6.811 x 1.12)
	$PVA_{12\%,15}$ = \$374,605	PVA_{due} = \$419,557.60
	Calculator solution: \$374,597.55	Calculator solution: \$419,549.25
C	$PVA_{20\%,9}$ = \$700 x 4.031	PVA_{due} = \$700 x (4.031 x 1.20)
	$PVA_{20\%,9}$ = \$2,821.70	PVA_{due} = \$3,386.04
	Calculator solution: \$2,821.68	Calculator solution: \$3,386.01

D $PVA_{5\%,7}$ = \$140,000 x 5.786 PVA_{due} = \$140,000 x (5.786 x 1.05)
$PVA_{5\%,7}$ = \$810,040 PVA_{due} = \$850,542
Calculator solution: \$810,092.28 Calculator solution: \$850,596.89

E $PVA_{10\%,5}$ = \$22,500 x 3.791 PVA_{due} = \$22,500 x (2.791 x 1.10)
$PVA_{10\%,5}$ = \$85,297.50 PVA_{due} = \$93,827.25
Calculator solution: \$85,292.70 Calculator solution: \$93,821.97

b. The annuity due results in a greater present value in each case. By depositing the payment at the beginning rather than at the end of the year, it has one less year to discount back.

4-20 LG 3: Ordinary Annuity versus Annuity Due

a. **Annuity C (Ordinary)** **Annuity D (Due)**
$FVA_{i\%,n}$ = PMT x ($FVIFA_{i\%,n}$) FVA_{due} = PMT x [$FVIFA_{i\%,n}$ x (1 + i)]

(1) $FVA_{10\%,10}$ = \$2,500 x 15.937 FVA_{due} = \$2,200 x (15.937 x 1.10)
$FVA_{10\%,10}$ = \$39,842.50 FVA_{due} = \$38,567.54
Calculator solution: \$39,843.56 Calculator solution: \$38,568.57

(2) $FVA_{20\%,10}$ = \$2,500 x 25.959 FVA_{due} = \$2,200 x (25.959 x 1.20)
$FVA_{20\%,10}$ = \$64,897.50 FVA_{due} = \$68,531.76
Calculator solution: \$64,896.71 Calculator solution: \$68,530.92

b. **(1)** At the end of year 10, at a rate of 10%, Annuity C has a greater value (\$39,842.50 vs. \$38,567.54).

(2) At the end of year 10, at a rate of 20%, Annuity D has a greater value (\$68,531.76 vs. \$64,896.71).

c. **Annuity C (Ordinary)** **Annuity D (Due)**
$PVA_{i\%,n}$ = PMT x ($FVIFA_{i\%,n}$) PVA_{due} = PMT x [$FVIFA_{i\%,n}$ x (1 + i)]

(1) $PVA_{10\%,10}$ = \$2,500 x 6.145 PVA_{due} = \$2,200 x (6.145 x 1.10)
$PVA_{10\%,10}$ = \$15,362.50 PVA_{due} = \$14,870.90
Calculator solution: \$15,361.42 Calculator solution: \$14,869.85

(2) $PVA_{20\%,10}$ = \$2,500 x 4.192 PVA_{due} = \$2,200 x (4.192 x 1.20)
$PVA_{20\%,10}$ = \$10,480 PVA_{due} = \$11,066.88
Calculator solution: \$10,481.18 Calculator solution: \$11,068.13

d. **(1)** At the beginning of the 10 years, at a rate of 10%, Annuity C has a greater value (\$15,362.50 vs. \$14,870.90).

(2) At the beginning of the 10 years, at a rate of 20%, Annuity D has a greater value (\$11,066.88 vs. \$10,480.00).

e. Annuity C, with an annual payment of \$2,500 made at the end of the year, has a higher present value at 10% than Annuity D with an annual payment of \$2,200 made at the beginning of the year. When the rate is increased to 20%, the shorter period of time to discount at the higher rate results in a larger value for Annuity D, despite the lower payment.

4-21 LG 3: Future Value of a Retirement Annuity

a.
FVA_{40} = \$2,000 x ($FVIFA_{10\%,40}$)
FVA_{40} = \$2,000 x (442.593)
FVA_{40} = \$885,186
Calculator solution: \$885,185.11

b.
FVA_{30} = \$2,000 x ($FVIFA_{10\%,30}$)
FVA_{30} = \$2,000 x (164.494)
FVA_{30} = \$328,988
Calculator solution: \$328,988.05

c. By delaying the deposits by 10 years the total opportunity cost is \$556,198. This difference is due to both the lost deposits of \$20,000 (\$2,000 x 10yrs.) and the lost compounding of interest on all of the money for 10 years.

d. **Annuity Due:**
FVA_{40} = \$2,000 x ($FVIFA_{10\%,40}$) x (1 + .10)
FVA_{40} = \$2,000 x (486.852)
FVA_{40} = \$973,704
Calculator solution: \$973,703.62

Annuity Due:
FVA_{30} = \$2,000 x ($FVIFA_{10\%,30}$) x (1.10)
FVA_{30} = \$2,000 x (180.943)
FVA_{30} = \$361,886
Calculator solution: \$361,886.85

Both deposits increased due to the extra year of compounding from the beginning-of-year deposits instead of the end-of-year deposits. However, the incremental change in the 40 year annuity is much larger than the incremental compounding on the 30 year deposit (\$88,518 versus \$32,898) due to the larger sum on which the last year of compounding occurs.

4-22 LG 3: Present Value of a Retirement Annuity

PVA = PMT x ($PVIFA_{9\%,25}$)
PVA = \$12,000 x (9.823)
PVA = \$117,876.00
Calculator solution: \$117,870.96

4-23 LG 3: Funding Your Retirement

a.
PVA = PMT x ($PVIFA_{11\%,30}$)
PVA = \$20,000 x (8.694)
PVA = \$173,880.00
Calculator solution: \$173,875.85

b.
PV = FV x ($PVIF_{9\%,20}$)
PV = \$173,880 x (.178)
PV = \$30,950.64
Calculator solution: \$31,024.82

c. Both values would be lower. In other words, a smaller sum would be needed in 20 years for the annuity and a smaller amount would have to be put away today to accumulate the needed future sum.

4-24 LG 2, 3: Present Value of an Annuity versus a Lump Sum

a.
PVA_n = PMT x ($PVIFA_{i\%,n}$)
PVA_{25} = \$40,000 x ($PVIFA_{5\%,25}$)
PVA_{25} = \$40,000 x 14.094

PVA_{25} = \$563,760
Calculator solution: \$563,757.78

At 5%, taking the award as an annuity is better; the present value is \$563,760, compared to receiving \$500,000 as a lump sum.

b. PVA_n = \$40,000 x ($PVIFA_{7\%, 25}$)
PVA_{25} = \$40,000 x (11.654)
PVA_{25} = \$466,160
Calculator solution: \$466,143.33

At 7%, taking the award as a lump sum is better; the present value of the annuity is only \$466,160, compared to the \$500,000 lump sum payment.

c. Because the annuity is worth more than the lump sum at 5% and less at 7%, try 6%:

PV_{25} = \$40,000 x ($PVIFA_{6\%,25}$)
PV_{25} = \$40,000 x 12.783
PV_{25} = \$511,320

The rate at which you would be indifferent is greater than 6%; about 6.25% Calculator solution: 6.24%

4-25 LG 3: Perpetuities: PVn **= PMT x ($PVIFA_{i\%,\infty}$)**

a.

Case	PV Factor
A	1 ÷ .08 = 12.50
B	1 ÷ .10 = 10.00
C	1 ÷ .06 = 16.67
D	1 ÷ .05 = 20.00

b. PMT x ($PVIFA_{i\%,\infty}$) = PMT x (1 ÷ i)

PMT		Factor		PV
\$20,000	x	12.50	=	\$ 250,000
\$100,000	x	10.00	=	\$1,000,000
\$3,000	x	16.67	=	\$ 50,000
\$60,000	x	20.00	=	\$1,200,000

4-26 LG 3: Creating an Endowment

a. PV = PMT x ($PVIFA_{i\%,\infty}$)
PV = (\$600 x 3) x (1 ÷ i)
PV = \$1,800 x (1 ÷ .06)
PV = \$1,800 x (16.67)
PV = \$30,006

b. PV = PMT x ($PVIFA_{i\%,\infty}$)
PV = (\$600 x 3) x (1 ÷ i)
PV = \$1,800 x (1 ÷ .09)
PV = \$1,800 x (11.11)
PV = \$19,998

4-27 LG 4: Future Value of a Mixed Stream
A.

Cash Flow Stream	Year	Number of Years to Compound	FV = CF x $FVIF_{12\%,n}$		Future Value
A	1	3	\$ 900 x 1.405	=	\$1,264.50
	2	2	1,000 x 1.254	=	1,254.00
	3	1	1,200 x 1.120	=	1,344.00
					\$3,862.50
			Calculator Solution:		\$3,862.84
B	1	5	\$ 30,000 x 1.762	=	\$52,860.00
	2	4	25,000 x 1.574	=	39,350.00
	3	3	20,000 x 1.405	=	28,100.00

	4	2	10,000 x 1.254	=	12,540.00'
	5	1	5,000 x 1.120	=	5,600.00
					$138,450.00
			Calculator Solution:		$138,450.79.
C	1	4	$ 1,200 x 1.574	=	$1,888.80
	2	3	1,200 x 1.405	=	1,686.00
	3	2	1,000 x 1.254	=	1,254.00
	4	1	1,900 x 1.120	=	2,128.00
					$6,956.80
			Calculator Solution:		$6,956.53

b. If payments are made at the beginning of each period the present value of each of the end-of-period cash flow streams will be multiplied by (1 + i) to get the present value of the beginning-of-period cash flows.

A $3,862.50 (1 + .12) = $4,326.00
B $138,450.00 (1 + .12) = $155,064.00
C $6,956.80 (1 + .12) = $7,791.62

4-28 LG 4: Future Value of Lump Sum versus a Mixed Stream

Lump Sum Deposit

FV_5 = PV x ($FVIF_{7\%,5)}$)
FV_5 = $24,000 X (1.403)
FV_5 = $33,672.00
Calculator solution: $33,661.24

Mixed Stream of Payments

Beginning of Year	Number of Years to Compound	FV = CF x $FVIF_{7\%,n}$		Future Value
1	5	$ 2,000 x 1.403	=	$2,806.00
2	4	$ 4,000 x 1.311	=	$5,244.00
3	3	$ 6,000 x 1.225	=	$7,350.00
4	2	$ 8,000 x 1.145	=	$9,160.00
5	1	$ 10,000 x 1.070	=	$10,700.00
				$35,260.00
		Calculator Solution:		$35,257.74

Gina should select the stream of payments over the front-end lump sum payment. Her future wealth will be higher by $1,588.

4-29 LG 4: Present Value-Mixed Stream

Cash Flow Stream	Year	CF	x	$PVIF_{12\%,n}$	=	Present Value
A	1	-$2000	x	.893	=	- $ 1,786
	2	3,000	x	.797	=	2,391
	3	4,000	x	.712	=	2,848
	4	6,000	x	.636	=	3,816

	5	8,000	x	.567	=	4,536
						$ 11,805
			Calculator solution			$ 11,805.51
B	1	$10,000	x	.893	=	$ 8,930
	2-5	5,000	x	2.712*	=	13,560
	6	7,000	x	.507	=	3,549
						$26,039
			Calculator solution:			$26,034.59

* Sum of PV factors for years 2 - 5

C	1-5	- $10,000	x	3.605*	$36,050
	6-10	8,000	x	2.045**	16,360
					$52,410
			Calculator Solution		$52,411.34

* PVIFA for 12% 5 years
** (PVIFA for 12%,10 years) - (PVIFA for 12%,5 years)

4-30 LG 4: Present Value-Mixed Stream

a.

Cash Flow Stream	Year	CF	x	$PVIF_{15\%,n}$	=	Present Value
A	1	$50,000	x	.870	=	$ 43,500
	2	40,000	x	.756	=	30,240
	3	30,000	x	.658	=	19,740
	4	20,000	x	.572	=	11,440
	5	10,000	x	.497	=	4,970
						$ 109,890
		Calculator solution				$ 109,856.33

Cash Flow Stream	Year	CF	x	$PVIF_{15\%,n}$	=	Present Value
B	1	$10,000	x	.870	=	$ 8,700
	2	20,000	x	.756	=	15,120
	3	30,000	x	.658	=	19,740
	4	40,000	x	.572	=	22,880
	5	50,000	x	.497	=	24,850
						$91,290
		Calculator solution				$91,272.98

b. Cash flow stream A, with a present value of $109,890, is higher than cash flow stream B's present value of $91,290 because the larger cash inflows occur in A in the early years when their present value is greater, while the smaller cash flows are received further in the future.

4-31 LG 1, 4: Present Value of a Mixed Stream

a.

Cash Flows

	$30,000	$25,000	$15,000	$15,000		$15,000	$10,000
0	1	2	3	4	•••	9	10

End of Year

b.

Cash Flow Stream	Year	CF	x	$PVIF_{12\%,n}$	=	Present Value
A	1	$30,000	x	.893	=	$ 26,790
	2	25,000	x	.797	=	19,925
	3-9	15,000	x	3.639*	=	54,585
	10	10,000	x	.322	=	3,220
						$ 104,520
			Calculator solution			$ 104,508.28

* The PVIF for this 7-year annuity is obtained by summing together the PVIFs of 12% for periods 3 through 9. This factor can also be calculated by taking the $PVIFA_{12\%,7}$ and multiplying by the $PVIF_{12\%,2}$.

c. Harte should accept the series of payments offer. The present value of that mixed stream of payments is greater than the $100,000 immediate payment.

4-32 LG 5: Funding Budget Shortfalls

a.

Year	Budget Shortfall	x	$PVIF_{8\%,n}$	=	Present Value
1	$5,000	x	.926	=	$ 4,630
2	4,000	x	.857	=	3,428
3	6,000	x	.794	=	4,764
4	10,000	x	.735	=	7,350
5	3,000	x	.681	=	2,043
					$ 22,215
			Calculator solution:		$22,214.03

A deposit of $22,215 would be needed to fund the shortfall for the pattern shown in the table.

b. An increase in the earnings rate would reduce the amount calculated in part **a**. The higher rate would lead to a larger interest being earned each year on the investment. The larger interest amounts will permit a decrease in the initial investment to obtain the same future value available for covering the shortfall.

4-33 LG 4: Relationship between Future Value and Present Value-Mixed Stream

a. Present Value

Year	CF	x	$PVIF_{5\%,n}$	=	Present Value
1	$ 800	x	.952	=	$ 761.60
2	900	x	.907	=	816.30
3	1,000	x	.864	=	864.00
4	1,500	x	.822	=	1,233.00
5	2,000	x	.784	=	1,568.00
					$5,242.90
			Calculator Solution:		$5,243.17

b. The maximum you should pay is $5,242.90.

c. A higher 7% discount rate will cause the present value of the cash flow stream to be lower than $5,242.90.

4-34 LG 5: Changing Compounding Frequency

(1) Compounding Frequency: $FV_n = PV \times FVIF_{i\%/m,\ n \times m}$

a. **Annual**
12 %, 5 years
FV_5 = $5,000 x (1.762)
FV_5 = $8,810
Calculator solution: $8,811.71

Semiannual
12% ÷ 2 = 6%, 5 x 2 = 10 periods
FV_5 = $5,000 x (1.791)
FV_5 = $8,955
Calculator solution: $8,954.24

Quarterly
12% ÷ 4 = 3%, 5 x 4 = 20 periods
FV_5 = $5,000 (1.806)
FV_5 = $9,030
Calculator solution: $9,030.56

b. **Annual**
16%, 6 years
FV_6 = $5,000 (2.436)
FV_6 = $12,180
Calculator solution: $12,181.98

Semiannual
16% ÷ 2 = 8%, 6 x 2 = 12 periods
FV_6 = $5,000 (2.518)
FV_6 = $12,590
Calculator solution: $12,590.85

Quarterly
16% ÷ 4 = 4%, 6 x 4 = 24 periods
FV_6 = $5,000 (2.563)
FV_6 = $12,815
Calculator solution: $12,816.52

c. **Annual**
20%, 10 years

Semiannual
20% ÷ 2 = 10%, 10 x 2 = 20 periods

$FV_{10} =$ $5,000 x (6.192)
$FV_{10} =$ $30,960
Calculator solution: $30,958.68

$FV_{10} =$ $5,000 x (6.727)
$FV_{10} =$ $33,635
Calculator solution: $33,637.50

Quarterly
20% ÷ 4 = 5%, 10 x 4 = 40 periods
$FV_{10} =$ $5,000 x (7.040)
$FV_{10} =$ $35,200
Calculator solution: $35,199.94

(2) Effective Interest Rate: $i_{eff} = (1 + i/m)^m - 1$

a. **Annual**
$i_{eff} = (1 + .12/1)^1 - 1$
$i_{eff} = (1.12)^1 - 1$
$i_{eff} = (1.12) - 1$
$i_{eff} = .12 = 12\%$

Semiannual
$i_{eff} = (1 + 12/2)^2 - 1$
$i_{eff} = (1.06)^2 - 1$
$i_{eff} = (1.124) - 1$
$i_{eff} = .124 = 12.4\%$

Quarterly
$i_{eff} = (1 + 12/4)^4 - 1$
$i_{eff} = (1.03)^4 - 1$
$i_{eff} = (1.126) - 1$
$i_{eff} = .126 = 12.6\%$

b. **Annual**
$i_{eff} = (1 + .16/1)^1 - 1$
$i_{eff} = (1.16)^1 - 1$
$i_{eff} = (1.16) - 1$
$i_{eff} = .16 = 16\%$

Semiannual
$i_{eff} = (1 + .16/2)^2 - 1$
$i_{eff} = (1.08)^2 - 1$
$i_{eff} = (1.166) - 1$
$i_{eff} = .166 = 16.6\%$

Quarterly
$i_{eff} = (1 + .16/4)^4 - 1$
$i_{eff} = (1.04)^4 - 1$
$i_{eff} = (1.170) - 1$
$i_{eff} = .170 = 17\%$

c. **Annual**
$i_{eff} = (1 + .20/1)^1 - 1$
$i_{eff} = (1.20)^1 - 1$
$i_{eff} = (1.20) - 1$
$i_{eff} = .20 = 20\%$

Semiannual
$i_{eff} = (1 + .20/2)^2 - 1$
$i_{eff} = (1.10)^2 - 1$
$i_{eff} = (1.210) - 1$
$i_{eff} = .210 = 21\%$

Quarterly
$i_{eff} = (1 + .20/4)^4 - 1$
$i_{eff} = (1.05)^4 - 1$
$i_{eff} = (1.216) - 1$
$i_{eff} = .216 = 21.6\%$

4-35 LG 5: Compounding Frequency, Future Value, and Effective Annual Rates

a. Compounding Frequency: $FV_n = PV \times FVIF_{i\%,n}$

A $FV_5 = \$2,500 \times (FVIF_{3\%,10})$
$FV_5 = \$2,500 \times (1.344)$
$FV_5 = \$3,360$
Calculator solution: $3,359.79

B $FV_3 = \$50,000 \times (FVIF_{2\%,18})$
$FV_3 = \$50,000 \times (1.428)$
$FV_3 = \$71,400$
Calculator solution: $71,412.31

C $FV_{10} = \$1,000 \times (FVIF_{5\%,10})$
$FV_{10} = \$1,000 \text{ X } (1.629)$
$FV_{10} = \$16,290$
Calculator solution: $1,628.89

D $FV_6 = \$20,000 \times (FVIF_{4\%,24})$
$FV_6 = \$20,000 \times (2.563)$
$FV_6 = \$51,260$
Calculator solution: $51,266.08

b. Effective Interest Rate: $i_{eff\,=}(1 + i\%/m)^{m} - 1$

A $i_{eff} = (1 + .06/2)^2 - 1$
$i_{eff} = (1 + .03)^2 - 1$
$i_{eff} = (1.061) - 1$
$i_{eff} = .061 = 6.1\%$

B $i_{eff} = (1 + .12/6)^6 - 1$
$i_{eff} = (1 + .02)^6 - 1$
$i_{eff} = (1.126) - 1$
$i_{eff} = .126 = 12.6\%$

C $i_{eff} = (1 + .05/1)^1 - 1$
$i_{eff} = (1 + .05)^1 - 1$
$i_{eff} = (1.05) - 1$
$i_{eff} = .05 = 5\%$

D $i_{eff} = (1 + .16/4)^4 - 1$
$i_{eff} = (1 + .04)^4 - 1$
$i_{eff} = (1.170) - 1$
$i_{eff} = .17 = 17\%$

c. The effective rates of interest rise relative to the stated nominal rate with increasing compounding frequency.

4-36 LG 2: Continuous Compounding: FVcont. = PV x e^x^ (e = 2.7183)

A $FV_{cont.} = \$1,000 \times e^{.18} = \$1,197.22$

B $FV_{cont.} = \$\,600 \times e^{1} = \$1,630.97$

C $FV_{cont.} = \$4,000 \times e^{.56} = \$7,002.69$

D $FV_{cont.} = \$2,500 \times e^{.48} = \$4,040.19$

Note: If calculator doesn't have e^x key, use y^x key, substituting 2.7183 for y.

4-37 LG 5: Compounding Frequency and Future Value

a. **(1)** $FV_{10} = \$2,000 \times (FVIF_{8\%,10})$
$FV_{10} = \$2,000 \times (2.159)$
$FV_{10} = \$4,318$
Calculator solution: $4,317.85

(2) $FV_{10} = \$2,000 \times (FVIF_{4\%,20})$
$FV_{10} = \$2,000 \times (2.191)$
$FV_{10} = \$4,382$
Calculator solution: $4,382.25

(3) $FV_{10} = \$2,000 \times (FVIF_{.022\%,3,600})$
$FV_{10} = \$2,000 \times (2.208)$
$FV_{10} = \$4,416$
Calculator solution: $4,415.23

(4) $FV_{10} = \$2,000 \times (e^{.8})$
$FV_{10} = \$2,000 \times (2.226)$
$FV_{10} = \$4,452$
Calculator solution: $4,451.08

b. **(1)** $i_{eff} = (1 + .08/1)^1 - 1$
$i_{eff} = (1 + .08)^1 - 1$
$i_{eff} = (1.08) - 1$
$i_{eff} = .08 = 8\%$

(2) $i_{eff} = (1 + .08/2)^2 - 1$
$i_{eff} = (1 + .04)^2 - 1$
$i_{eff} = (1.082) - 1$
$i_{eff} = .082 = 8.2\%$

(3) $i_{eff} = (1 + .08/360)^{360} - 1$
$i_{eff} = (1 + .00022)^{360} - 1$
$i_{eff} = (1.0824) - 1$
$i_{eff} = .0824 = 8.24\%$

(4) $i_{eff} = (e^k - 1)$
$i_{eff} = (e^{.08} - 1)$
$i_{eff} = (1.0833 - 1)$
$i_{eff} = .0833 = 8.33\%$

c. Compounding continuously will result in $134 more dollars at the end of the 10 year period than compounding annually.

d. The more frequent the compounding the larger the future value. This result is shown in part a by the fact that the future value becomes larger as the compounding period moves from annually to continuously. Since the future value is larger for a given fixed amount invested, the effective return also increases directly with the frequency of compounding. In part b we see this fact as the effective rate moved from 8% to 8.33% as compounding frequency moved from annually to continuously.

4-38 LG 5: Comparing Compounding Periods

a. $\mathbf{FV_n}$ **PV x** $\mathbf{FVIF_{i\%,n}}$

(1) **Annually:** $FV = PV \times FVIF_{12\%,2} = \$15,000 \times (1.254) = \$18,810$
Calculator solution: $18,816

(2) **Quarterly:** $FV = PV \times FVIF_{3\%,8} = \$15,000 \times (1.267) = \$19,005$
Calculator solution: $19,001.55

(3) **Monthly:** $FV = PV \times FVIF_{1\%,24} = \$15,000 \times (1.270) = \$19,050$
Calculator solution: $19,046.02

(4) **Continuously:** $FV_{cont.} = PV \times e^{xt}$
$FV = PV \times 2.7183^{.24} = \$15,000 \times 1.27125 = \$19,068.77$
Calculator solution: $19,068.74

b. The future value of the deposit increases from $18,810 with annual compounding to $19,068.77 with continuous compounding, demonstrating that future value increases as compounding frequency increases.

c. The maximum future value for this deposit is $19,068.77, resulting from continuous compounding, which assumes compounding at every possible interval.

4-39 LG 3, 5: Annuities and Compounding: $\mathbf{FVA_n = PMT \times (FVIFA_{i\%,n})}$

a.

(1) Annual
$FVA_{10} = \$300 \times (FVIFA_{8\%,10})$
$FVA_{10} = \$300 \times (14.487)$
$FVA_{10} = \$4,346.10$
Calculator solution: = $4,345.97

(2) Semiannual
$FVA_{10} = \$150 \times (FVIFA_{4\%,20})$
$FVA_{10} = \$150 \times (29.778)$
$FVA_{10} = \$4,466.70$
Calculator Solution: $4,466.71

(3) Quarterly
$FVA_{10} = \$75 \times (FVIFA_{2\%,40})$
$FVA_{10} = \$75 \times (60.402)$
$FVA_{10} = \$4,530.15$
Calculator solution: $4,530.15

b. The sooner a deposit is made the sooner the funds will be available to earn interest and contribute to compounding. Thus, the sooner the deposit and the more frequent the compounding, the larger the future sum will be.

4-40 LG 6: Deposits to Accumulate Growing Future Sum: $PMT = \dfrac{FVA_n}{FVIFA_{i\%,\,n}}$

Case	Terms	Calculation		Payment
A	12%, 3 yrs.	PMT = \$5,000 ÷ 3.374	=	\$ 1,481.92
		Calculator solution:		\$ 1,481.74
B	7%, 20 yrs.	PMT = \$100,000 ÷ 40.995	=	\$ 2,439.32
		Calculator solution:		\$ 2,439.29
C	10%, 8 yrs.	PMT = \$30,000 ÷ 11.436	=	\$ 2,623.29
		Calculator solution:		\$ 2,623.32
D	8%, 12 yrs.	PMT = \$15,000 ÷ 18.977	=	\$ 790.43
		Calculator solution:		\$ 790.43

4-41 LG 6: Creating a Retirement Fund

a. $PMT = FVA_{42} \div (FVIFA_{8\%,42})$
$PMT = \$220{,}000 \div (304.244)$
$PMT = \$723.10$

b. $FVA_{42} = PMT \times (FVIFA_{8\%,42})$
$FVA_{42} = \$600 \times (304.244)$
$FVA_{42} = \$182{,}546.40$

4-42 LG 6: Accumulating a Growing Future Sum

$FV_n = PV \times (FVIF_{i\%,n})$
$FV_{20} = \$85{,}000 \times (FVIF_{6\%,20})$
$FV_{20} = \$85{,}000 \times (3.207)$
$FV_{20} = \$272{,}595$ = Future value of retirement home in 20 years.
Calculator solution: \$ 272,606.52

$PMT = FV \div (FVIFA_{i\%,n})$
$PMT = \$272{,}595 \div (FVIFA_{10\%,20})$
$PMT = \$272{,}595 \div (57.274)$
$PMT = \$4{,}759.49$
Calculator solution: \$4,759.61 = annual payment required.

4-43 LG 3, 5: Deposits to Create a Perpetuity

a. Present value of a perpetuity = PMT x (1 ÷ i)
= \$6,000 x (1 ÷ .10)
= \$6,000 x 10
= \$60,000

b. $PMT = FVA \div (FVIFA_{10\%,10})$
$PMT = \$60{,}000 \div (15.937)$
$PMT = \$ 3{,}764.82$
Calculator solution: \$ 3,764.72

4-44 LG 2, 3, 6: Inflation, Future Value, and Annual Deposits

a. $FV_n = PV \times (FVIF_{i\%,n})$
$FV_{20} = \$200{,}000 \times (FVIF_{5\%,25})$
$FV_{20} = \$200{,}000 \times (3.386)$

FV_{20} = $677,200 = Future value of retirement home in 25 years.
Calculator solution: $ 677,270.99

b. PMT = FV ÷ ($FVIFA_{i\%,n}$)
PMT = $677,200 ÷ ($FVIFA_{9\%,25}$)
PMT = $677,200 ÷ (84.699)
PMT = $7,995.37
Calculator solution: $7,995.19 = annual payment required.

c. Since John will have an additional year on which to earn interest at the end of the 25 years his annuity deposit will be smaller each year. To determine the annuity amount John will first discount back the $677,200 one period.

$$PV_{24} = \$677{,}200 \times .9174 = \$621{,}263.28$$

John can solve for his annuity amount using the same calculation as in part b.

PMT = FV ÷ ($FVIFA_{i\%,n}$)
PMT = $621,263.78 ÷ ($FVIFA_{9\%,25}$)
PMT = $621,263.78 ÷ (84.699)
PMT = $7,334.95
Calculator solution: $7,334.78 = annual payment required.

4-45 LG 6: Loan Payment: $\text{PMT} = \dfrac{\text{PVA}}{\text{PVIFA}_{i\%,\,n}}$

Loan

A
PMT = $12,000 ÷ ($PVIFA_{8\%,3}$)
PMT = $12,000 ÷ 2.577
PMT = $4,656.58
Calculator solution: $4,656.40

B
PMT = $60,000 ÷ ($PVIFA_{12\%,10}$)
PMT = $60,000 ÷ 5.650
PMT = $10,619.47
Calculator solution: $10,619.05

C
PMT = $75,000 ÷ ($PVIFA_{10\%,30}$)
PMT = $75,000 ÷ 9.427
PMT = $7,955.87
Calculator Solution: $7,955.94

D
PMT = $4,000 ÷ ($PVIFA_{15\%,5}$)
PMT = $4,000 ÷ 3.352
PMT = $1,193.32
Calculator solution: $1,193.26

4-46 LG 6: Loan Amortization Schedule

a. PMT = $15,000 ÷ ($PVIFA_{14\%,3}$)
PMT = $15,000 ÷ 2.322
PMT = $6,459.95
Calculator solution: $6,460.97

b.

End of Year	Loan Payment	Beginning of Year Principal	Payments Interest	Payments Principal	End of Year Principal
1	$ 6,459.95	$15,000.00	$2,100.00	$4,359.95	$10,640.05
2	$ 6,459.95	10,640.05	1,489.61	4,970.34	5,669.71
3	$ 6,459.95	5,669.71	793.76	5,666.19	0

(The difference in the last year's beginning and ending principal is due to rounding.)

c. Through annual end-of-the-year payments, the principal balance of the loan is declining, causing less interest to be accrued on the balance.

4-47 LG 6: Loan Interest Deductions

a.
PMT = \$10,000 ÷ ($PVIFA_{13\%,3}$)
PMT = \$10,000 ÷ (2.361)
PMT = \$4,235.49
Calculator solution: \$4,235.22

b.

End of Year	Loan Payment	Beginning of Year Principal	**c.** Payments Interest	Payments Principal	End of Year Principal
1	$ 4,235.49	$ 10,000.00	$ 1,300.00	$ 2,935.49	$ 7,064.51
2	4,235.49	7,064.51	918.39	3,317.10	3,747.41
3	4,235.49	3,747.41	487.16	3,748.33	0

(The difference in the last year's beginning and ending principal is due to rounding.)

4-48 LG 6: Monthly Loan Payments

a.
PMT = \$4,000 ÷ ($PVIFA_{1\%,24}$)
PMT = \$4,000 ÷ (21.243)
PMT = \$188.28
Calculator solution: \$188.29

b.
PMT = \$4,000 ÷ ($PVIFA_{.75\%,24}$)
PMT = \$4,000 ÷ (21.889)
PMT = \$182.74
Calculator solution: \$182.74

4-49 LG 6: Growth Rates

a. PV = FV_n x $PVIF_{i\%,n}$

Case

A
PV = FV_4 x $PVIF_{k\%,4yrs.}$
\$500 = \$800 x $PVIF_{k\%,4yrs}$
.625 = $PVIF_{k\%,4yrs}$
12% < k < 13%
Calculator Solution: 12.47%

B
PV = FV_9 x $PVIF_{i\%,9yrs.}$
\$1,500 = \$2,280 x $PVIF_{k\%,9yrs.}$
.658 = $PVIF_{k\%,9yrs.}$
4%<k<5%
Calculator solution: 4.76%

C
PV = FV_6 x $PVIF_{i\%,6}$

b.

Case

A Same

B Same

C Same

$\$2,500 = \$2,900 \times PVIF_{k\%,6\ yrs.}$
$.862 = PVIF_{k\%,6yrs.}$
$2\% < k < 3\%$
Calculator solution: 2.50%

c. The growth rate and the interest rate should be equal, since they represent the same thing.

4-50 LG 6: Rate of Return: $PV_n = FV_n \times (PVIF_{i\%,n})$

a. $PV = \$2,000 \text{ X } (PVIF_{i\%,3yrs.})$
$\$1,500 = \$2,000 \times (PVIF_{i\%,3\ yrs.})$
$.75 = PVIF_{i\%,3\ yrs.}$
$10\% < i < 11\%$
Calculator solution: 10.06%

b. Mr. Singh should accept the investment that will return $2,000 because it has a higher return for the same amount of risk.

4-51 LG 6: Rate of Return and Investment Choice

a.

A
$PV = \$8,400 \times (PVIF_{i\%,6yrs.})$
$\$5,000 = \$8,400 \times (PVIF_{i\%,6\ yrs.})$
$.595 = PVIF_{i\%,6\ yrs.}$
$9\% < i < 10\%$
Calculator solution: 9.03%

B
$PV = \$15,900 \times (PVIF_{i\%,15yrs.})$
$\$5,000 = \$15,900 \times (PVIF_{i\%,15yrs.})$
$.314 = PVIF_{i\%,15yrs.}$
$8\% < i < 9\%$
Calculator solution: 8.02%

C
$PV = \$7,600 \times (PVIF_{i\%,4yrs.})$
$\$5,000 = \$7,600 \times (PVIF_{i\%,4\ yrs.})$
$.658 = PVIF_{i\%,4\ yrs.}$
$11\% < i < 12\%$
Calculator solution: 11.04%

D
$PV = \$13,000 \times (PVIF_{i\%,10\ yrs.})$
$\$5,000 = \$13,000 \times (PVIF_{i\%,10\ yrs.})$
$.385 = PVIF_{i\%,10\ yrs..}$
$10\% < i < 11\%$
Calculator solution: 10.03%

b. Investment C provides the highest return of the 4 alternatives. Assuming equal risk for the alternatives, Clare should choose C.

4-52 LG 6: Rate of Return-Annuity: $PVA_n = PMT \times (PVIFA_{i\%,n})$
$\$10,606 = \$2,000 \times (PVIFA_{i\%,10\ yrs.})$
$5.303 = PVIFA_{i\%,10\ yrs.}$
$13\% < i < 14\%$
Calculator solution: 13.58%

4-53 LG 6: Choosing the Best Annuity: $PVA_n = PMT \times (PVIFA_{i\%,n})$

a.

Annuity A
$\$30,000 = \$3,100 \times (PVIFA_{i\%,20\ yrs.})$
$9.677 = PVIFA_{i\%,20\ yrs.}$
$8\% < i < 9\%$
Calculator solution: 8.19%

Annuity B
$\$25,000 = \$3,900 \times (PVIFA_{i\%,10\ yrs.})$
$6.410 = PVIFA_{i\%,10\ yrs.}$
$9\% < i < 10\%$
Calculator solution: 9.03%

Annuity C

Annuity D

$\$40,000 = \$4,200 \times (PVIFA_{i\%,15\ yrs.})$	$\$35,000 = \$4,000 \times (PVIFA_{i\%,12\ yrs.})$
$9.524 = PVFA_{i\%,15\ yrs.}$	$8.75 = PVIFA_{i\%,12\ yrs.}$
$6\% < i < 7\%$	$5\% < i < 6\%$
Calculator solution: 6.3%	Calculator solution: 5.23%

b. Loan B gives the highest rate of return at 9% and would be the one selected based upon Raina's criteria.

4-54 LG 6: Interest Rate for an Annuity

a. Defendants interest rate assumption

$\$2,000,000 = \$156,000 \times (PVIFA_{i\%,25\ yrs.})$
$12.821 = PVFA_{i\%,25\ yrs.}$
$5\% < i < 6\%$
Calculator solution: 5.97%

b. Prosecution interest rate assumption

$\$2,000,000 = \$255,000 \times (PVIFA_{i\%,25\ yrs.})$
$7.843 = PVFA_{i\%,25\ yrs.}$
$i = 12\%$
Calculator solution: 12.0%

c. $\$2,000,000 = PMT \times (PVIFA_{9\%,25yrs.})$
$\$2,000,000 = PMT\ (9.823)$
$PMT = \$203,603.79$

4-55 LG 6: Loan Rates of Interest: $PVA_n = PMT \times (PVIFA_{i\%,n})$

a.

Loan A	**Loan B**
$\$5,000 = \$1,352.81 \times (PVIFA_{i\%,5\ yrs.})$	$\$5,000 = \$1,543.21 \times (PVIFA_{i\%,4\ yrs.})$
$3.696 = PVIFA_{i\%,5\ yrs.}$	$3.24 = PVIFA_{i\%,\ 4\ yrs.}$
$i = 11\%$	$i = 9\%$

Loan C

$\$5,000 = \$2,010.45 \times (PVIFA_{i\%,3\ yrs.})$
$2.487 = PVIFA_{k\%,3\ yrs.}$
$i = 10\%$

b. Mr. Fleming should choose Loan B, which has the lowest interest rate.

4-56 LG 6: Number of Years – Single Amounts

A	**B**
$FV = PV \times (FVIF_{7\%,n\ yrs.})$	$FV = \$12,000 \times (FVIF_{5\%,n\ yrs.})$
$\$1,000 = \$300 \times (FVIF_{7\%,n\ yrs.})$	$\$15,000 = \$12,000 \times (FVIF_{5\%,n\ yrs.})$
$3.333 = FVIF_{7\%,n\ yrs.}$	$1.250 = FVIF_{5\%,n\ yrs.}$
$17 < n < 18$	$4 < n < 5$
Calculator solution: 17.79	Calculator solutions: 4.573

C	**D**
$FV = PV \times (FVIF_{10\%,n\ yrs.})$	$FV = \$100 \times (FVIF_{9\%,n\ yrs.})$
$\$20,000 = \$12,000 \times (FVIF_{10\%,n\ yrs.})$	$\$500 = \$100 \times (FVIF_{9\%,n\ yrs.})$
$1.667 = FVIF_{10\%,n\ yrs.}$	$5.00 = FVIF_{9\%,n\ yrs.}$
$5 < n < 6$	$18 < n < 19$

Calculator solution: 5.36

Calculator solution: 18.68

E

FV = PV x $(FVIF_{15\%,n\ yrs.})$

\$30,000 = \$7,500 x $(FVIF_{15\%,n\ yrs.})$

4.000 = $FVIF_{15\%,n\ yrs.}$

$9 < n < 10$

Calculator solution: 9.92

4-57 LG 6: Time to Accumulate a Given Sum

a. 20,000 = \$10,000 x $(FVIF_{10\%,n\ yrs.})$

2.000 = $FVIF_{10\%,n\ yrs.}$

$7 < n < 8$

Calculator solution: 7.27

b. 20,000 = \$10,000 x $(FVIF_{7\%,n\ yrs.})$

2.000 = $FVIF_{7\%,n\ yrs.}$

$10 < n < 11$

Calculator solution: 10.24

c. 20,000 = \$10,000 x $(FVIF_{12\%,n\ yrs.})$

2.000 = $FVIF_{12\%,n\ yrs.}$

$6 < n < 7$

Calculator solution: 6.12

d. The higher the rate of interest the less time is required to accumulate a given future sum.

4-58 LG 6: Number of Years – Annuities

A

PVA = PMT x $(PVIFA_{11\%,n\ yrs.})$

\$1,000 = \$250 x $(PVIFA_{11\%,n\ yrs.})$

4.000 = $PVIFA_{11\%,n\ yrs.}$

$5 < n < 6$

Calculator solution: 5.56

B

PVA = PMT x $(PVIFA_{15\%,n\ yrs.})$

\$150,000 = \$30,000 x $(PVIFA_{15\%,n\ yrs.})$

5.000 = $PVIFA_{15\%,n\ yrs.}$

$9 < n < 10$

Calculator solution: 9.92

C

PVA = PMT x $(PVIFA_{10\%,n\ yrs.})$

\$80,000 = \$30,000 x $(PVIFA_{10\%,n\ yrs.})$

2.667 = $PVIFA_{10\%,n\ yrs.}$

$3 < n < 4$

Calculator solution: 3.25

D

PVA = PMT x $(PVIFA_{9\%,n\ yrs.})$

\$600 = \$275 x $(PVIFA_{9\%,n\ yrs.})$

2.182 = $PVIFA_{9\%,n\ yrs.}$

$2 < n < 3$

Calculator solutions: 2.54

E

PVA = PMT x $(PVIFA_{6\%,n\ yrs.})$

\$17,000 = \$3,500 x $(PVIFA_{6\%,n\ yrs.})$

4.857 = $PVIFA_{6\%,n\ yrs.}$

$5 < n < 6$

Calculator solution: 5.91

4-59 LG 6: Time to Repay Installment Loan

a. $\$14{,}000 = \$2{,}450 \times (PVIFA_{12\%,n\ yrs.})$
$5.714 = PVIFA_{12\%,n\ yrs.}$
$10 < n < 11$
Calculator solution: 10.21

b. $\$14{,}000 = \$2{,}450 \times (PVIFA_{9\%,n\ yrs.})$
$5.714 = PVIFA_{9\%,n\ yrs.}$
$8 < n < 9$
Calculator solution: 8.37

c. $\$14{,}000 = \$2{,}450 \times (PVIFA_{15\%,n\ yrs.})$
$5.714 = PVIFA_{15\%,n\ yrs.}$
$13 < n < 14$
Calculator solution: 13.92

d. The higher the interest rate the greater the number of time periods needed to repay the loan fully.

Chapter 4 Case

Finding Jill Moran's Retirement Annuity

Chapter 4's case challenges the student to apply present and future value techniques to a real-world situation. The first step in solving this case is to determine the total amount Sunrise Industries needs to accumulate until Ms. Moran retires, remembering to take into account the interest that will be earned during the 20-year payout period. Once that is calculated, the annual amount to be deposited can be determined.

a.

Cash inflow: Accumulation Period	**Cash outflow: Distribution Period**
12 end-of-year deposits; Earns interest at 9%	20 end-of-year payments of $42,000 balance earns interest at 12%

|——————————|——————————————————————|>
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32

End of Year

b. **Total amount to accumulate by end of year 12**

PV_n = PMT x ($PVIFA_{i\%,n}$)
PV_{20} = \$42,000 x ($PVIFA_{12\%,20}$)
PV_{20} = \$42,000 x 7.469
PV_{20} = \$313,698
Calculator solution: $313,716.63

c. **End-of-year deposits, 9% interest:** $PMT = \dfrac{FVA_n}{FVIFA_{i\%,\,n}}$

PMT = \$313,698 ÷ ($FVIFA_{9\%,\ 12\ yrs.}$)
PMT = $313,698 ÷ 20.141
PMT = $15,575.10
Calculator solution: $15,575.31

Sunrise Industries must make a $15,575.10 annual end-of-year deposit in years 1-12 in order to provide Ms. Moran a retirement annuity of $42,000 per year in years 13 to 32.

d. **End-of-year deposits, 10% interest**

PMT = \$313,698 ÷ ($FVIFA_{10\%,12\ yrs.}$)
PMT = $313,698 ÷ 21.384
PMT = $14,669.75
Calculator solution: $14,669.56

The corporation must make a $14,669.75 annual end-of-year deposit in years 1-12 in order to provide Ms. Moran a retirement annuity of $42,000 per year in years 13 to 32.

e. **Initial deposit if annuity is a perpetuity and initial deposit earns 9%:**

PV_{perp} = PMT x (1 ÷ i)
PV_{perp} = \$42,000 x (1 ÷ .12)
PV_{perp} = \$42,000 x 8.333
PV_{perp} = \$349,986

End-of-year deposit:
PMT = FVA_n ÷ ($FVIFA_{i\%,n}$)

PMT = $349,986 ÷ (FVIFA$_{9\%,12\ yrs.}$)
PMT = $349,986 ÷ 20.141
PMT = $17,376.79
Calculator solution: 17,377.04